臺灣綜合大學系統112學年度學士班轉學生聯合招生考試試題

科目名稱	微積分B	類組代碼	共同考科
		科目碼	E0012

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 1 頁

Show all your work and justify all your answers.

1. (a) (5 points) Evaluate $\lim_{x\to 2}\dfrac{x^2+5}{x^3-2x}$.

   (b) (5 points) Evaluate $\displaystyle\int_{-1}^{1} 6x^5+3x^2\,dx$.

2. (10 points) Evaluate $\displaystyle\int_1^2 \frac{1}{x^2}\cos\left(\frac{\pi}{x}\right)dx$.

3. (10 points) Evaluate $\lim_{x\to-\infty}\dfrac{\ln(8^x+9^x)}{3x}$.

4. (10 points) Find an equation for the tangent line to the curve $x^2+4xy+y^3+5=0$ at the point $(2,-1)$.

5. (10 points) Let $f(x)=\dfrac{x}{(x-1)^2}$. Find the intervals on which f is increasing and the intervals on which f is decreasing.

6. (10 points) Evaluate $\displaystyle\int_0^1 \frac{1}{2x+10\sqrt{x}+12}\,dx$.

7. (10 points) Evaluate $\displaystyle\int_0^{\pi/2}\int_y^{2y}\frac{\sin x}{x}\,dxdy+\int_{\pi/2}^{\pi}\int_y^{\pi}\frac{\sin x}{x}\,dxdy$.

8. (10 points) Use the method of Lagrange multipliers to find the minimum value of the function
$$f(x,y,z)=x-y+z$$
subject to the constraint $2x^2+4y^2+4z^2=9$.

9. (10 points) Let $f(x,y,z)=x^2ze^y+xz^2$. In what direction does f increase most rapidly at the point $(1,\ln 2,2)$? What is the maximum rate of increase?

10. (10 points) For $|x|<\dfrac{3}{2}$, the function $f(x)=\dfrac{2x^5}{16x^4+81}$ can be expressed as a power series $\displaystyle\sum_{n=0}^{\infty}a_nx^n$, where a_n's are constants. Find the value of a_{25}.